Hidden non-locality

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1 Introduction

The article [1] makes important contributions to the nonlocal nature of quantum mechanics and how this nature can be demonstrated experimentally. The following questions will help better understand the basis of the subject:

1. What is the connection between the concepts of quantum entanglement and Bell nonlocality?
   When a particle with zero spin splits into two, their total spin remains zero. Measuring one particle affects the other's state instantly, demonstrating entanglement. The EPR paradox [2] claims this violates locality and suggests hidden variables. Bell's experiments proved no local hidden variables exist, demonstrating Bell nonlocality. Bell inequalities set limits on correlations; fully entangled states can exceed these limits, violating Bell's inequality.
2. Do all entangled states lead to Bell inequality violation?
   If the entanglement is weak or the measurements are not ideal, it does not lead to a violation of Bell's inequalities. For instance, situations like partially entangled Werner states or weakening of entanglement due to decoherence may not lead to a violation of Bell's inequalities.
3. What is local filtering, and how should it be applied in the context of a test of Bell's inequality?
   Local filtering, is a process used in quantum information theory that allows the extraction of specific outcomes from measurements on a quantum state [3]. To test of Bell's inequality and reveal hidden nonlocality, the required steps are explained in chapter 4.
4. What does it mean that certain entangled state feature "hidden nonlocality"
   After applying operations such as local filtering to two entangled particles that initially do not violate Bell's inequality, these parts may subsequently violate Bell's inequality. This demonstrates that a system initially presenting local behavior can exhibit a nonlocal nature through appropriate manipulations. This phenomenon is known as hidden nonlocality.

2 Nonlocality

Bell's inequality [4] states that any local hidden variable theory must satisfy a specific inequality that sets a limit on the correlations between entangled particles. This inequality is given by:

$$|E(A,B) - E(A,B') + E(A',B) + E(A',B')| \leq 2$$

where $E(A,B)$ is the expectation value of the product of the outcomes when measurements A and B are performed on the two particles.

2.1 Local Hidden Variable Theories

In local hidden variable theories, the outcome of a measurement on one particle is determined by preexisting properties (hidden variables) and is not influenced by measurements performed on a distant particle. Mathematically, this can be expressed as:

$$P(a,b|\lambda) = P(a|\lambda)P(b|\lambda)$$

where $P(a,b|\lambda)$ is the joint probability of outcomes a and b given hidden variable λ.

2.2 Quantum Mechanical Predictions

Quantum mechanics states that entangled particles can violate Bell's inequality, leading to nonlocality, which implies that the measurement on one particle can instantaneously influence the state of another particle regardless of the distance separating them.

The proposed paper focus on classify mixed states into local or nonlocal, as for pure states the problem is completely solved [5].

3 Werner Density Matrix

The density matrices used in the main article were introduced by Werner[1]. He proposed a density matrix W that, while not decomposable into a direct product of pure states, did not violate standard Bell inequality. Under a single ideal measurement, this state could be described by hidden variables models, stated explicitly by Werner.

Popescu uses the following Werner state for dimensions $d \geq 5$:

$$W = \frac{1}{d^2}\left(\frac{1}{d}I^{d\times d} + 2\Sigma_{i<j}\,|S_{ij}\rangle\langle S_{ij}|\right)$$

Where

$$|S_{ij}\rangle = \frac{1}{\sqrt{2}}(|i,j\rangle - |j,i\rangle)$$

denotes the "spin 1/2 singlet" state.

4 Proposed Measurement

To reveal hidden nonlocality, the following measurement sequence is proposed:

1. Preparation of initial states: An entangled quantum state is created that is shared between Alice and Bob (ie. Bell states).
2. Filtering process: After making initial measurements on their own qubits, Alice and Bob apply local filtering. The filtered density matrix is defined as:
$$\tilde{\rho} = \frac{1}{N}\left[(F_A \otimes F_B)\rho(F_A^\dagger \otimes F_B^\dagger)\right]$$
where $N = Tr\left[(F_A \otimes F_B)\rho(F_A^\dagger \otimes F_B^\dagger)\right]$ is a normalization factor, and F_A and F_B are positive operators acting on $\mathbb{C}^d$ representing the local filtering of Alice and Bob [6].
3. Measurements: Further measurements on the filtered state are used to verify the success of the filtering process and to evaluate the extent to which the resulting quantum state violates the Bell inequality.

In this example, The author uses P and Q as initial measurements, where P and Q are defined as:

$$P = |1\rangle_{11}\langle 1| + |2\rangle_{11}\langle 2|$$
$$Q = |1\rangle_{22}\langle 1| + |2\rangle_{22}\langle 2|$$

Then, he performs new measurements, either A or A' for the first particle and B or B' for the second particle. All four operators have three eigenvalues: 1, -1, and 0. 1 and -1 have corresponding eigenvectors in subspaces $|1\rangle_1, |2\rangle_1$ and $|1\rangle_2, |2\rangle_2$, respectively. As for eigenvalue 0, it is highly degenerate and corresponds to every other subspace. Also, operators A, A', B, and B' are such operators that maximally violate the CHSH inequality for the singlet state:

$$\langle S_{12}|\,AB + AB' + A'B - A'B'\,|S_{12}\rangle = 2\sqrt{2}$$

By splitting the initial ensemble into subensembles based on the outcomes of the initial projection measurements, the author shows that the correlations in one of these subensembles can violate the CHSH inequality. Specifically, for dimensions $d \geq 5$, the state W' formed after the initial projections violates the CHSH inequality, demonstrating nonlocality:

$$W' = \frac{1}{N}(P \otimes Q)W(Q^\dagger \otimes P^\dagger)$$
$$W' = \frac{2d}{2d+4}\left(\frac{1}{2d}I^{(2\times 2)} + |S_{12}\rangle\langle S_{12}|\right)$$

$$\mathrm{Tr}(W'(AB+AB'+A'B-A'B')) = \frac{2d}{2d+4}\cdot 2\sqrt{2} \geq 2$$

5 Conclusion

Determining the locality of quantum states is more challenging than it initially appears. The paper has demonstrated that certain states exhibit hidden nonlocality, which can only be revealed through various different measurements. A single ideal measurement is insufficient to detect locality; local states must not violate Bell inequality, or any similar inequalities, even after a sequence of non-ideal measurements. Only when the correlations between the results of any local experiments on the state can be described by a local hidden variables model can we definitively state that the state has no hidden nonlocality.

References

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